Math 1: Calculus with Algebra **Quiz 1** Name: Answer Key
Fall 2015

Instructions: (points) This quiz consists of 5 problems covering material from the first week of class. Credit is awarded for **correct solutions** in which you **show your work**. You will have 30 minutes to complete this quiz.

For this quiz, we will consider the following functions:

$$\boxed{f(x) = \sqrt{x+4}, \qquad g(x) = x^2 - 4, \qquad h(x) = \frac{1}{3}x}.$$

(5^{pts}) **1.** Evaluate the following expressions:

(a) $(f+g)(5)$

Solution: By definition of function addition, $(f+g)(5) = f(5)+g(5) = \sqrt{5+4}+(5^2-4) = 3+(25-4) = 3+21 = 24$.

(b) $(f \cdot g)(0)$

Solution: By definition of function multiplication, $(f \cdot g)(0) = f(0) \cdot g(0) = \sqrt{4}(0^2-4) = 2(-4) = -8$.

(c) $\left(\frac{g}{f}\right)(3)$

Solution: By definition of function quotients, $\left(\frac{g}{f}\right)(3) = \frac{g(3)}{f(3)} = \frac{3^2-4}{\sqrt{3+4}} = \frac{9-4}{\sqrt{7}} = \frac{5}{\sqrt{7}}$.

(5^{pts}) **2.** Complete each of the following:

(a) Determine which of the following sequences are the same as the sequence $\{0, 1, 0, 1, \ldots\}$.

(A) $\{1+(-1)^n\}_{n=1}^{\infty}$ (B) $\{-1+(-1)^n\}_{n=0}^{\infty}$

(✔) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$ (D) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$

(b) Write out the first 5 terms of the sequence $\{3n+2\}_{n=2}^{\infty}$.

Solution: The first 5 terms are: $8, 11, 14, 17, 20$.

(5^{pts}) **3.** Use the functions from the beginning of the quiz to complete each of the following:

(a) Find the domain of f.

Solution: The domain of f is $[-4, \infty)$.

(b) Find the range of f.

Solution: The range of f is $[0, \infty)$.

(c) Find the domain of $\frac{f}{h}$.

Solution: The value h is 0 at $x = 0$ and the domain of h is $\mathbb{R}$. From part (a), f is defined on $[-4, \infty)$. So the function $\frac{f}{h}$ has domain $[-4, 0) \cup (0, \infty)$.

(5^{pts}) **4.** Use the functions from the beginning of the quiz to find *and simplify* the following expressions:

(a) $(f \circ h)(x)$

Solution: By definition, $(f \circ h)(x) = f(h(x)) = f\left(\frac{1}{3}x\right) = \sqrt{\frac{1}{3}x + 4}$.

(b) $\left(\frac{f}{g}\right)(x)$

Solution: By definition, $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+4}}{x^2-4}$.

(c) $(f \circ g)(x)$

Solution: By definition, $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = \sqrt{(x^2 - 4) + 4} = \sqrt{x^2} = |x|$.

(4^{pts}) **5.** *True* or *False*. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

(a) **F** If k is a function, then $k(a + b) = k(a) + k(b)$.

Solution: Consider the function $k(x) = x^2$ and let $a = 1$ and $b = 1$. Then $k(1+1) = k(2) = 2^2 = 4$ but $k(1) + k(1) = 1^2 + 1^2 = 2$.

(b) **F** If k is a function and $k(a) = k(b)$ then $a = b$.

Solution: Consider the function $k(x) = x^2$ and let $a = -2$ and $b = 2$. Then $k(a) = (-2)^2 = 4$ and $k(b) = 2^2 = 4$. So $k(a) = k(b)$ but $a \neq b$.

(c) **T** A vertical line intersects the graph of a function at most once.

Solution: This is the content of the Vertical Line Test.

(d) **F** If x is any real number then $\sqrt{x^2} = x$.

Solution: If $x < 0$ then $\sqrt{x^2} = -x$. For example, if $x = -1$ then $\sqrt{x^2} = \sqrt{(-1)^2} = \sqrt{1} = 1 = -x$.